

# Low Complexity Single-carrier ZF-frequency-domain equalization for Space frequency block-coded transmissions over Nakagami $m$ fading channels

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**Abstract** - In this paper, we combine the two techniques like space-frequency block-coding (SFBC) with single-carrier frequency-domain equalization (SC-FDE) in wireless Nakagami fading channels where channel response is not same for adjacent subcarriers. Nakagami -  $m$  fading channel exploit the varies fading environment for different values of fading parameter  $m$ . Nakagami -  $m$  distribution approximates a Hoyt or Nakagami -  $q$  distribution for  $m$  in the range  $m \leq 1$  and a Ricean fading for the fading parameter  $m$  in the range  $1 < m < \infty$  while for  $m=1$  it is act as a Rayleigh Distribution. In general view Receiver produces error when the matched filter is used and error floor in bit error rate performance. To avoid this, combine receiver is derived under a low complexity zero forcing (ZF) which has less computational complexity for each symbol compare to classical zero forcing.

**Keyword:** Single-carrier frequency-domain equalization (SC-FDE), space-frequency block-code (SFBC), cyclic prefix (CP).

## I. INTRODUCTION

In single-carrier communication system equalization can be done either in the time domain or in the frequency domain, which has studied in (Crochiere and Rabiner 1983; Imand Kang 2001). Single-carrier frequency-domain equalization (SC-FDE) was shown in (Clark 1998a; Sari, Karam and Jeanclaude 1995; Liu, Giannakis, Scaglione and Barbarossa 1999a) to have attractive equalization scheme for broadband wireless fading channels which are given by their lengthen impulse response memory. Under these conditions, single-carrier zero forcing frequency-domain equalization (SC ZF-FDE) has lower complexity, due to its use of the computationally-efficient fast Fourier transform (FFT), than time-domain equalization whose complexity increase with channel memory and spectral efficiency (trellis-based

schemes) or require very long FIR filters. The SC-FDE was shown in (Liu, Giannakis, Scaglione and Barbarossa 1999a) to have two main advantages over orthogonal frequency division multiplexing (OFDM), first one is reduced sensitivity to carrier frequency errors and lower peak-to-average ratio (PAR). The single-carrier frequency-domain equalization (SC-FDE) and orthogonal frequency division multiplexing (OFDM) have similarity in performance and structure while unpleasant condition the nonlinear distortion and carrier synchronization problems inherent to OFDM.

A simple and powerful diversity technique using two transmit antennas was first proposed by Alamouti (Alamouti 1998b). Space-time block-coding (STBC) scheme has been proposed in several wireless applications due to its many attractive features (Alamouti 1998b). First one, at full transmission rate, it achieves full spatial diversity for any signal constellation with 2 transmit antennas and. Second, it does not require channel side information at the transmitter. Third, maximum likelihood decoding of STBC done by simple linear processing. There has been some work on combining the Alamouti's STBC scheme with OFDM (Alamouti 1998b) and with time-domain equalization (Lindskog, and Paulraj 2000a). Combining STBC with SC-FDE was first presented by Al-Dhahir which use the benefits of both schemes (Al-Dhahir 2001). The STBC scheme is suitable for slowly varying channels because the channel is assumed to be constant for two consecutive symbol intervals (Alamouti 1998b; Bauch 2003; Lee and Williams 2000b).

To mitigate effect of fast fading caused by high-speed mobile environment, we propose an SFBC SC-FDE system. In STBC we apply codes across two antennas over

two adjacent time interval whereas in SFBC codes apply across two antennas over two adjacent subcarriers and thus becomes effective in solving the problem of fast fading distortion in frequency non selective fading mobile environment (Falconer, Ariyavisitakul, Benyamin-Seeyar and Eidson2002; Lindskog, and Paulraj 2000). This SFBC system has been also applied for OFDM systems (Al-Dhahir 2001). The SC system is designed so we get frequency and spatial diversities at the transmitter. The combining receiver is derived under a low complexity zero forcing (ZF) criterion. Unlike in OFDM-based system, the proposed system cannot achieve full spatial diversity and multipath, because it does not use a maximum likelihood (ML) decoding (Bauch 2003). However, the ZF equalizer used in the proposed system offers a low computational complexity.

## II. SFBC SC-FDE system

### 2.1 Transmit diversity and Channel model

We assume SC system with two transmit antennas and one receive antenna. The transmit sequence in SC system is in time domain so we cannot apply SFBC system directly to an SC system. The SC transmit sequence given in (Jang, Won and Im 2006) is equivalent to the SFBC. The transmitted signal of the first antenna can be by,

$$\begin{aligned} x_1(m) &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_1(k) W_N^{-mk} \\ &= \frac{1}{\sqrt{2}} [x^e(m) + W_N^{-m} x^o(m)] \end{aligned} \quad (1)$$

Where  $m=0, 1, 2, \dots, N-1$  and  $x_1(m)$  is original signal to be transmitted from antenna 1.

$$x^e(m) = \sqrt{\frac{2}{N}} \sum_{v=0}^{(N/2)-1} X_1(2v) W_{N/2}^{-vm} \quad (2)$$

$$x^o(m) = \sqrt{\frac{2}{N}} \sum_{v=0}^{(N/2)-1} X_1(2v+1) W_{N/2}^{-vm} \quad (3)$$

Similarly,

$$x_2(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_2(k) W_N^{-mk}$$

$$\begin{aligned} &= \frac{1}{\sqrt{N}} \sum_{v=0}^{(N/2)-1} -X_1^*(2v) + W_N^{-n} X_1^*(2v+1) \times W_{N/2}^{-mv} \\ &= \frac{1}{\sqrt{2}} [-x^{o*}(-m) + W_N^{-n} x^{e*}(-m)] \end{aligned} \quad (4)$$

Where  $m=0, 1, 2, \dots, N-1$  and  $x_2(m)$  is original signal to be transmitted from antenna 2.

We assume single-carrier symbol transmission over an additive-noise frequency-selective channel with memory  $N$ . Each block of length  $N$  is appended in head of with a length- $N_p$  cyclic prefix to reduce effect of inter block interference (IBI) and channel matrices become circulant.

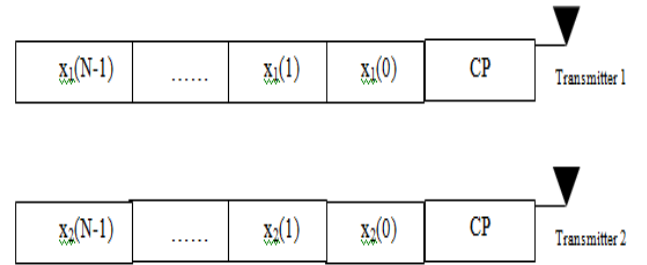


Figure 1. Transmit scheme

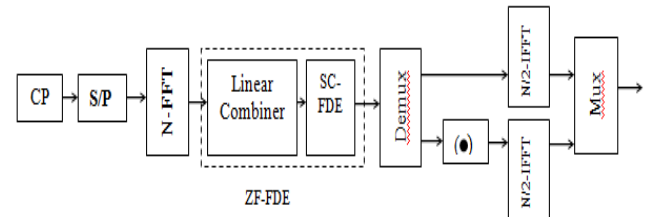


Figure 2. ZF-receiver

### 2.2 Proposed ZF Combining Receiver

Figure 2 shows a block diagram of the combining receiver with low complexity zero forcing (LZF) technique. The transmitted signal  $x_i$ , where  $i=1, 2$ , the received block  $r$  after removing the first received  $N+N_p$  symbols corresponding to the cyclic prefix. Hence, out of every received symbols, only  $N$  symbols are processed. The input-output relationship can be expressed in matrix form as follows,

$$r = h_1 x_1 + h_2 x_2 + n \quad (5)$$

where  $h_i$ ,  $i=1, 2$ , are the circulant channel matrices from the first and second transmit antennas, respectively, and  $n$  is the zero-mean complex additive white Gaussian noise (AWGN) with length  $N$  symbols. The channel matrix  $h_i$  is  $N \times N$  right circulant with first column is the channel impulse response (CIR). Since Eigen decompose of  $h_i$  is given by,

$$h_i = \varphi \rho_i \varphi^H \quad (6)$$

where  $\varphi^H$  denotes complex conjugate transpose of  $\varphi$  is the twiddle or orthonormal discrete Fourier transform (DFT) matrix.  $\rho_i$  is  $N \times N$  diagonal matrix whose diagonal elements are channel frequency response.

By taking DFT of Equation (5) we get received signal as,

$$\begin{aligned} R &= \rho_1 \varphi x_1 + \rho_2 \varphi x_2 + N' \\ &= \rho_1 X_1 + \rho_2 X_2 + N' \end{aligned} \quad (7)$$

Equation (7) is written in matrix form as,

$$\begin{aligned} R'_n &= \begin{bmatrix} R_n \\ R_{n+1}^* \end{bmatrix} \\ &= \begin{bmatrix} \rho_{1,n} & -\rho_{2,n} \\ \rho_{2,n+1}^* & \rho_{1,n+1}^* \end{bmatrix} \begin{bmatrix} X_{1,n} \\ X_{1,n+1}^* \end{bmatrix} + \begin{bmatrix} N_n \\ N_{n+1}^* \end{bmatrix} \\ &= \rho'_n X'_n + N'_n \end{aligned} \quad (8)$$

$\rho_{i,n}$  represent the channel frequency response (CFR) of the  $n^{\text{th}}$  subcarrier between the  $i^{\text{th}}$  transmit and receive antenna. Note that the channel coefficients of adjacent subcarriers are not equal. Applying matched filter (MF) to (8) to obtain the estimated symbols as

$$\begin{aligned} Y'_n &= \rho_n'^H R'_n \\ &= \begin{bmatrix} K_n & \varepsilon \\ \varepsilon & K_{n+1} \end{bmatrix} \begin{bmatrix} X_{1,n} \\ X_{1,n+1}^* \end{bmatrix} + \begin{bmatrix} N_n \\ N_{n+1}^* \end{bmatrix} \end{aligned} \quad (9)$$

where diversity gain  $K_n = |\rho_{1,n}|^2 + |\rho_{2,n}|^2$ ,  $K_{n+1} = |\rho_{1,n+1}|^2 + |\rho_{2,n+1}|^2$  and  $\varepsilon = \rho_{2,n+1} \rho_{1,n+1}^* - \rho_{1,n}^* \rho_{2,n}$  is the interference term. To remove the interference produce by the MF receiver and provide diversity gains as in the classical ZF receiver, we propose low complexity zero forcing (ZF) matrix is given by:

$$\rho_n^{LZF} = \begin{bmatrix} \rho_{1,n}^* \rho_{2,n+1} / G_n \\ \rho_{2,n}^* \rho_{1,n+1} / G_n^* \end{bmatrix} \quad (10)$$

Where  $G_n = \rho_{2,n+1} \rho_{1,n+1}^* / \rho_{1,n}^* \rho_{2,n}$ , now we can derive a linear combination receiver under the ZF criterion.

$$\begin{aligned} Y_n^{LZF} &= \rho_n^{LZF} R'_n \\ &= \begin{bmatrix} K'_n & 0 \\ 0 & K'_{n+1} \end{bmatrix} \begin{bmatrix} X_{1,n} \\ X_{1,n+1}^* \end{bmatrix} + \begin{bmatrix} N'_n \\ N'_{n+1} \end{bmatrix} \end{aligned} \quad (11)$$

Here  $K'_n = |\rho_{1,n}|^2 + |\rho_{2,n}|^2 / G_n$  and  $K'_{n+1} = |\rho_{1,n+1}|^2 + |\rho_{2,n+1}|^2 / G_n^*$ .

$$\begin{aligned} \widetilde{X}_n &= (\rho_n^{LZF} \rho'_n)^{-1} Y_n^{LZF} \\ &= \begin{bmatrix} \widetilde{X}_n \\ \widetilde{X}_{n+1}^* \end{bmatrix} \end{aligned} \quad (12)$$

In the SC-FDE detection is made in the time domain whereas channel equalization is performed in the frequency domain. Therefore, the estimated transmitted symbols can be obtained as by taking  $N/2$  point DFT of  $\widetilde{X}_n$  and  $\widetilde{X}_{n+1}^*$ .

The proposed ZF receivers given in (12) remove the effect of interference from adjacent subcarriers while providing diversity gains  $K'_n$  and  $K'_{n+1}$ . This method avoids the inversion of the transfer matrix of channel which has half computational complexity for each symbol compare to classical zero forcing for SFBC SC-FDE.

### III. SIMULATION RESULTS

The performance of bit-error rate for two transmit antenna and one received antenna (SFBC SC-FDE) system was investigated through computer simulation. We consider Nakagami fading channel using Jakes model ( $f_{dt_s} = 0.01$ ) with QPSK modulation, and a symbol duration of 3.69s as in the proposed third generation TDMA cellular standard EDGE (Furuskar, Mazur, Muller and Olofsson 1999b). The proposed low complexity ZF receiver give better performance in bit error rate than MF receiver and it has less computational complexity for each symbol while the degradation in performance at  $10^{-3}$  is 0.1dB than the classical ZF receiver.

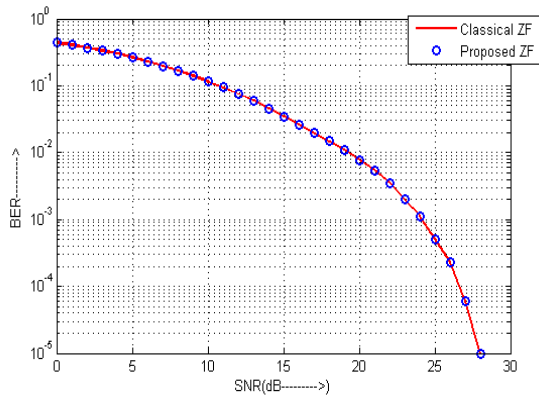


Figure 3. BER of SFBC SC ZF-FDE for time varying Nakagami Fading channel with QPSK modulation and  $N = 64$ .

#### IV. CONCLUSION AND FUTURE WORK

The SFBC and SC ZF-FDE reduce complexity than SFBC-OFDM. It has been shown that the proposed LZF receiver achieves approximately identical performance as classical ZF decreasing computational complexity. Further it will be extended for space time frequency block code (STFBC).

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